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# Observational Cosmology

R.H. Sanders

Kapteyn Astronomical Institute, Groningen, The Netherlands

**Abstract.** I discuss the classical cosmological tests— angular size-redshift, flux-redshift, and galaxy number counts— in the light of the cosmology prescribed by the interpretation of the CMB anisotropies. The discussion is somewhat of a primer for physicists, with emphasis upon the possible systematic uncertainties in the observations and their interpretation. Given the curious composition of the Universe inherent in the emerging cosmological model, I stress the value of searching for inconsistencies rather than concordance, and suggest that the prevailing mood of triumphalism in cosmology is premature.

## 1 Introduction

The traditional cosmological tests appear to have been overshadowed by observations of the anisotropies in the cosmic microwave background (CMB). We are told that these observations accurately measure the geometry of the Universe, its composition, its present expansion rate, and the nature and form of the primordial fluctuations [1]. The resulting values for these basic parameters are very similar to those deduced earlier from a variety of observations—the so-called “concordance model”— with about 30% of the closure density of the Universe comprised of matter (mostly a pressureless, non-baryonic dark matter), the remainder being in negative pressure dark energy [2]. Given the certainty and precision of these assertions, any current discussion of observational cosmology must begin with the question: Is there any room for doubt? Why should we bother with lower precision cosmological tests when we know all of the answers anyway?

While the interpretation of the CMB anisotropies has emerged as the single most important cosmological tool, we must bear in mind that the conclusions drawn do rest upon a number of assumptions, and the results are not altogether as robust as we are, at times, led to believe. One such assumption, for example, is that of adiabatic initial fluctuations— that is, 100% adiabatic. A small admixture of correlated isocurvature fluctuations, an aspect of braneworld scenarios [3], can affect peak amplitudes and thus, the

derived cosmological parameters. A more fundamental assumption is that of the validity of traditional Friedmann-Robertson-Walker (FRW) cosmology in the post-decoupling universe. Is the expansion of the universe described by the Friedmann equation? Even minimal changes to the right-hand-side, such as the equation of state of the dark energy component, can alter the angular size distance to the last scattering surface at  $z=1000$  and the luminosity distance to distant supernovae. But even more drastic changes to the Friedmann equation, resulting from modified gravitational physics, have been proposed in attempts to remove the unattractive dark energy [4, 5].

Such suggestions reflect a general unease with the concordance model— a model that presents us with a universe that is strange in its composition. The most abundant form of matter consists of, as yet, undetected non-baryonic particles originally postulated to solve the problems of structure formation and of the missing mass in bound gravitational systems such as galaxies and clusters of galaxies. In this second respect, it is fair to say that it has failed— or, to be generous, not yet succeeded— because the predicted density distribution of dark halos which emerge from cosmic N-body [6] simulations appears to be inconsistent with observations of spiral galaxies [7] or with strong lensing in clusters of galaxies [8].

Even more mysterious is the “dark energy”, the pervasive homogeneous fluid with a negative pressure which may be identified with the cosmological constant, the zero-point energy density of the vacuum. The problem of this unnaturally low energy density,  $10^{-122}$  in Planck units, is well-known, as is the cosmic coincidence problem: why are we observing the Universe at a time when the cosmological constant has, fairly recently, become dynamically important [9]? To put it another way, why are the energy densities of matter and dark energy so comparable at the present epoch? This is strange because the density of matter dilutes with the expanding volume of the Universe while the vacuum energy density does not. It is this problem which has led to the proposal of dynamic dark energy, quintessence— a dark energy, possibly associated with a light scalar field— with an energy density that evolves with cosmic time possibly tracking the matter energy density [10]. Here the difficulty is that the field would generally be expected to have additional observational consequences— such as violations of the equivalence principle at some level, possibly detectable in fifth force experiments [9].

For these reasons, it is even more important to pursue cosmological tests that are independent of the CMB, because one might expect new physics to appear as observations inconsistent with the concordance model. In this sense, discord is more interesting than concord; to take a Hegelian point of view— ideas progress through dialectic, not through concordance. It is with this in mind that I will review observational cosmology with emphasis upon CMB-independent tests.

Below I argue that the evolution of the early, pre-recombination universe is well-understood and tightly constrained by considerations of primordial nucleosynthesis. If one wishes to modify general relativity to give deviations

from Friedmann expansion, then such modifications are strongly constrained at early times, at energies on the order of 1 MeV. However, cosmological evolution is much less constrained in the post-recombination universe where there is room for deviation from standard Friedmann cosmology and where the more classical tests are relevant. I will discuss three of these classical tests: the angular size distance test where I am obliged to refer to its powerful modern application with respect to the CMB anisotropies; the luminosity distance test and its application to observations of distant supernovae; and the incremental volume test as revealed by faint galaxy number counts.

These classical tests yield results that are consistent, to lower precision, with the parameters deduced from the CMB. While one can make minimal changes to standard cosmology, to the equation of state of the dark energy for example, which yield different cosmological parameters, there is no compelling observational reason to do so. It remains the peculiar composition and the extraordinary coincidences embodied by the concordance model that call for deeper insight. Such motivations for questioning a paradigm are not unprecedented; similar worries led to the inflationary scenario which, unquestionably, has had the dominant impact on cosmological thought in the past 25 years and which has found phenomenological support in the recent CMB observations.

I am not going to discuss cosmological tests based upon specific models for structure formation, such as the form of the luminous matter power spectrum [11] or the amplitude of the present mass fluctuations [12]. I do not mean to imply that such tests are unimportant, it is only that I restrict myself here to more global and model-independent tests. If one is considering a possibility as drastic as a modification of Friedmann expansion due, possibly, to new gravitational physics, then it is tests of the global curvature and expansion history of the Universe that are primary.

I am also going to refrain, in so far as possible, from discussion of theory—of new gravitational physics or of any other sort. The theoretical issues presented by dark matter that can only be detected gravitationally or by an absurdly small but non-zero cosmological constant are essentially not problems for the interpretive astronomer. The primary task is to realistically assess the reliability of conclusions drawn from the observations, and that is what I intend to do.

## 2 Astronomy made simple (for physicists)

I think that it is fair to assume that most of you are physicists, so I begin by defining some of the units and terminology used by astronomers. I do this because much of this terminology is arcane for those not in the field.

First of all there is the peculiar logarithmic scale of flux—magnitudes—whereby a factor of 100 in flux is divided into five equal logarithmic intervals. The system is ancient and has its origin in the logarithmic response of the

human eye. The ratio of the flux of two objects is then given by a difference in magnitudes; i.e.,

$$m_2 - m_1 = -2.5 \log(F_2/F_1) \quad (2.1)$$

where, one will notice, smaller magnitude means larger flux. The zero-point of this logarithmic scale is set by some standard star such as Vega. Because this is related to the flux, and not the luminosity of an object, it is called the “apparent” magnitude. Distant galaxies have apparent magnitudes, in visible light, of greater than 20, and the galaxies in the Hubble Deep Field, go down to magnitudes of 30. The magnitude is typically measured over a specified wavelength range or color band, such as blue (B), visual (V), or infrared (K), and these are designated  $m_B$ ,  $m_V$ , and  $m_K$ , or sometimes just B,V, and K. This is made more confusing by the fact that there are several competing photometric systems (or sets of filters) and conversion between them is not always simple.

With a particular photometric system one can measure the color of an astronomical object, expressed as difference in magnitudes in two bands, or color index; e.g.,

$$B - V = 2.5 \log(F_V/F_B) \quad (2.2)$$

Here a larger B-V color index means that an object is relatively redder; a smaller B-V that the object is bluer. Unlike the apparent magnitude, this is an intrinsic property of the object. Or rather, it is intrinsic once the astronomer corrects the magnitudes in the various bands to the zero-redshift ( $z = 0$ ) frame. This is called the “K-correction” and requires a knowledge of the intrinsic spectral energy distribution (SED) of the source, be it a galaxy or a distant supernova.

The luminosity of an object is also an intrinsic property and is usually expressed by astronomers as an “absolute” magnitude. This is the apparent magnitude an object would have if it were placed at a standard distance, taken to be 10 parsecs, i.e.  $3 \times 10^{17}$  m (more on parsecs below). Because this distance is small by extragalactic standards the absolute magnitudes of galaxies turn out to be rather large negative numbers:  $M_G \approx -18$  to  $-21$ . The luminosity of a galaxy  $L_G$  in units of the solar luminosity  $L_\odot$  can be determined from the relation

$$M_G - M_\odot = -2.5 \log(L_G/L_\odot) \quad (2.3)$$

where the absolute magnitude of the sun (in the V band) is 5.5. The luminosities of galaxies typically range from  $10^8$  to  $10^{11}$   $L_\odot$ . The peak absolute magnitude of a type I supernova (SNIa) is about -19.5, or comparable to an entire galaxy. This is one reason why these objects are such ideal extragalactic distance probes.

The unit of distance used by astronomers is also archaic: the parsec which is about  $3 \times 10^{16}$  m or about 3 light years. This is the distance to a star with an semi-annual parallax of 1 arc second and is not a bad unit when one

is discussing the very local region of the galaxy. Our galaxy has a diameter between 10 to 20 kiloparsecs, so the kiloparsec is an appropriate unit when discussing galactic structure. The appropriate unit of extragalactic distance, however, is the “megaparsec” or Mpc, with nearby galaxies being those at distances less than 10 Mpc. The nearest large cluster of galaxies, the Virgo cluster, is at a distance of 20 Mpc, and very distant galaxies are those further than 100 Mpc, although here one has to be careful about how distance is operationally defined.

We all know that the Universe is uniformly expanding and the Hubble parameter,  $H$ , is the recession velocity of galaxies per unit distance, with  $H_o$  being its value in the present Universe. It is typically measured in units of  $\text{km s}^{-1}\text{Mpc}^{-1}$  or inverse time. A number of observations point to  $H_o \approx 70 \text{ km s}^{-1}\text{Mpc}^{-1}$ . The Hubble time is defined as  $t_H = H_o^{-1}$  which is about  $9.8 \times 10^9 h^{-1}$  years, and this must be comparable to the age of the Universe. The definition  $h = H_o/100 \text{ km s}^{-1}\text{Mpc}^{-1}$  is a relic of the recent past when the Hubble parameter was less precisely determined, but I keep using it below because it remains convenient as a unit-less quantity. We can also define a characteristic scale for the universe which is the Hubble radius or  $r_H = c/H_o$  and this is  $3000 h^{-1} \text{ Mpc}$ . This would be comparable to the “distance” to the horizon.

Just for interest, one could also define a Hubble acceleration or  $a_H = cH_o \approx 7 \times 10^{-10} \text{ m/s}^2$ . This modest acceleration of 7 angstroms/second squared is, in effect, the acceleration of the Hubble flow at the horizon if we live in a Universe dominated by a cosmological constant as observations seem to suggest. It is also comparable to the acceleration in the outer parts of galaxies where the need for dark matter first becomes apparent [13]. In some sense, it is remarkable that such a small acceleration has led to a major paradigm shift.

### 3 Basics of FRW cosmology

The fundamental assumption underlying the construction of cosmological models is that of the cosmological principle: The Universe appears spatially isotropic in all its properties to all observers. The only metric which is consistent with this principle is the Robertson-Walker metric:

$$ds^2 = c^2 dt^2 - \frac{a^2(t) dr^2}{[1 - r^2/R_o^2]} - a^2(t) r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \quad (3.1)$$

where  $r$  is the radial comoving coordinate,  $a(t)$  is the dimensionless scale factor by which all distances vary as a function of cosmic time, and  $R_o^{-2}$  is a parameter with dimensions of inverse length squared that describes the curvature of the Universe and may be positive, zero, or negative (see [14] for a general discussion).

This is the geometry of the Universe, but dynamics is provided by General Relativity– the Einstein field equations– which yield ordinary differential equations for  $a(t)$ . The time-time component leads to a second order equation:

$$\ddot{a} = -\frac{4\pi G}{3}a(\rho + 3p/c^2) \quad (3.2)$$

where  $\rho$  is the density,  $p$  is the pressure and the quantity in parenthesis is the active gravitational mass density. Considering conservation of energy for a perfect fluid

$$d(\rho V) = -pdV/c^2 \quad (3.3)$$

with an equation of state

$$p = w\rho c^2 \quad (3.4)$$

we have  $\rho \propto a^{-1(1+w)}$ . The equation of state combined with eq. 3.2 tells us that the Universe is accelerating if  $w < -1/3$ .

The space-space components combined with the time-time component yield the usual first-order Friedmann equation

$$\left(\frac{H}{H_o}\right)^2 - \frac{\Omega_k}{a^2} = \sum_i \Omega_i a^{-3(1+w_i)} \quad (3.5)$$

where  $H = \dot{a}/a$  is the running Hubble parameter, the summation is over the various fluids comprising the Universe and

$$\Omega_i = \frac{8\pi G\rho_i}{3H_o^2} \quad (3.6)$$

with  $\Omega_k = -(r_H/R_o)^2$ . We often see eq. 3.5 written in terms of redshift where  $a = (1+z)^{-1}$ . Each component has its own equation of state parameter,  $w_i$ :  $w = 0$  for non-relativistic matter (baryons, CDM);  $w = 1/3$  for radiation or other relativistic fluid;  $w = -1$  for a cosmological constant; and  $-1 < w < -1/3$  for “quintessence”, dynamic dark energy resulting in ultimate acceleration of the universal expansion. I will not consider  $w < -1$  which has been termed “phantom” dark energy [15]; here the effective density increases as the Universe expands (this could be realized by a ghost field, a scalar with a kinetic term in the Lagrangian having the wrong sign so it rolls up rather than down a potential hill).

Given a universe composed of radiation, non-relativistic matter, and quintessence, the Friedmann equation takes its familiar form:

$$\left(\frac{H}{H_o}\right)^2 - \frac{\Omega_k}{a^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_Q a^{-3(1+w)}. \quad (3.7)$$

Here it is evident that radiation drives the expansion at early times ( $a \ll 1$ ), non-relativistic matter at later times, a non-vanishing curvature ( $\Omega_k \neq 0$ ) at later times still, and, if  $w < -1/3$ , the vacuum energy density ultimately

dominates. For the purpose of this lecture, I refer to eq. 3.7 with  $w = -1$  (the usual cosmological constant) as standard FRW cosmology, while  $0 > w \neq -1$  would represent a minimal modification to FRW cosmology. Moreover, when  $w = -1$ , I replace  $\Omega_Q$  by  $\Omega_\Lambda$ . I will not consider changes to the Friedmann equation which might result from modified gravitational physics.

Because the subject here is observational cosmology we must discuss the operational definitions of distance in an FRW Universe. If there exists a standard meter stick, an object with a known fixed linear size  $d$  which does not evolve with cosmic time, then one could obviously define an angular size distance:

$$D_A = \frac{d}{\theta} \quad (3.8)$$

where  $\theta$  would be the observed angle subtended by this object. If there exists a standard candle, an object with a known fixed luminosity  $L$  which does not vary with cosmic time, then one could also define a luminosity distance:

$$D_L = \left( \frac{L}{4\pi F} \right)^{\frac{1}{2}} \quad (3.9)$$

where  $F$  is the measured flux of radiation.

For a RW universe both the angular size distance and the luminosity distance are related to the radial comoving coordinate,

$$r = |R_o| \chi \left[ \frac{r_H}{|R_o|} \int_{\tau_o}^{\tau} \frac{d\tau}{a(\tau)} \right] \quad (3.10)$$

where  $\tau = tH_o$ ,  $R_o^2 = -r_H^2/\Omega_k$ , and

$$\chi(x) = \sin(x) \quad \text{if } \Omega_k < 0$$

$$\chi(x) = \sinh(x) \quad \text{if } \Omega_k > 0$$

$$\chi(x) = x \quad \text{if } \Omega_k = 0.$$

Then it is the case that

$$D_A = r a(\tau) = r/(1+z) \quad (3.11a)$$

and

$$D_L = r/a(\tau) = r(1+z). \quad (3.11b)$$

It is evident that both the angular size distance and the luminosity distance depend upon the expansion history (through  $\int d\tau/a(\tau)$ ) and the curvature (through  $\chi(x)$ ).

The same is true of a comoving volume element:

$$dV = r^2 dr d\Omega \quad (3.12)$$

where here  $d\Omega$  is an incremental solid angle. Therefore, if there exists a class of objects with a non-evolving comoving density, then this leads to another

possible cosmological test: simply count those objects as a function of redshift or flux.

Below, I am going to consider these measures of distance and volume in the form of three classical cosmological tests:

1. Angular size tests which essentially involve the determination of  $D_A(z)$ . Here one measure  $\theta$  for objects with a known and (hopefully) standard linear size (such as compact radio sources).
2. Luminosity distance tests which involve the measurement of  $F(z)$  for presumably standard candles (such as supernova type Ia, SNIa).
3.  $dV/dz$  test which involve the counts of very faint galaxies as a function of flux and redshift.

But before I come to these classic tests, I want to discuss the evidence supporting the validity of the standard hot Big Bang, as an appropriate description of the early pre-recombination Universe.

## 4 Observational support for the standard model of the early Universe

The discovery 40 years ago of the cosmic microwave background radiation (CMB) ended, for most people, the old debate about Steady-State vs. the Hot Big Bang. Ten years ago, support for the Hot Big Bang was fortified by the COBE satellite which demonstrated that the CMB has a Planck spectrum to extremely high precision; it is, quite literally, the most perfect black body observed in nature [16]. This makes any model in which the CMB is produced by some secondary process, such as thermal re-radiation of starlight by hot dust, seem extremely difficult, if not impossible, to contrive.

Not only does the background radiation have a thermal spectrum, it is now evident that this radiation was hotter in the past than now as expected for adiabatic expansion of the Universe. This is verified by observations of neutral carbon fine structure lines as well as molecular hydrogen rotational transitions in absorption line systems in the spectra of distant quasars. Here, the implied population of different levels, determined primarily by the background radiation field, is an effective thermometer for that radiation field. One example is provided by a quasar with an absorption line system at  $z = 3.025$  which demonstrates that the temperature of the CMB at this redshift was  $12.1^{+1.7}_{-8.2}$  K, consistent with expectations ( $T \propto 1 + z$ ) [17].

However, the most outstanding success story for the Hot Big Bang is generally considered to be that of Big Bang Nucleosynthesis (BBN) which, for a given number of relativistic particle species, predicts the primordial abundances of the light isotopes with, effectively, one free parameter: the ratio of baryons-to-photons,  $\eta$  [18]. I want to review this success story, and point out that there remains one evident inconsistency which may be entirely observational, but which alternatively may point to new physics.



We saw above in the Friedmann equation (eq. 3.7) that radiation, if present, will always dominate the expansion of the Universe at early enough epochs (roughly at  $z \approx 2 \times 10^4 \Omega_m$ .) This makes the expansion and thermal history of the Universe particularly simple during this period. The Friedmann equation becomes

$$H^2 = \frac{4\pi G a T^4 N(T)}{3c^2}; \quad (4.1)$$

here  $a$  is the radiation constant and  $N(T)$  is the number of degrees of freedom in relativistic particles. The scale factor is seen to grow as  $t^{1/2}$  which means that the age of the Universe is given by  $t = 1/2H$ . This implies, from eq. 4.1, an age-temperature relation of the form  $t \propto T^{-2}$ . Putting in numbers, the precise relation is

$$t = \frac{2.5}{T_{MeV}^2 N(T)^{\frac{1}{2}}} \text{ s} \quad (4.2)$$

where the age is given in seconds and  $T_{MeV}$  is the temperature measured in MeV. It is only necessary to count the number of relativistic particle species:

$$N(T) = \sum g_B + \frac{7}{8} \sum g_F \quad (4.3)$$

where the sums are over the number of bosonic degrees of freedom ( $g_B$ ) and fermionic degrees of freedom ( $g_F$ ). The factor  $7/8$  is due to the difference in Bose-Einstein and Fermi-Dirac statistics. Adding in all the known species—photons, electrons-positrons (when  $T_{MeV} > 0.5$ ), three types of neutrinos and anti-neutrinos— we find

$$t \approx T_{MeV}^{-2} \text{ s} \quad (4.4)$$

for the age-temperature relation in the early Universe.

When the Universe is less than one second old ( $T > 1$  MeV) the weak interactions

$$p + e^- \leftrightarrow n + \nu_e$$

$$n + e^+ \leftrightarrow p + \nu_e$$

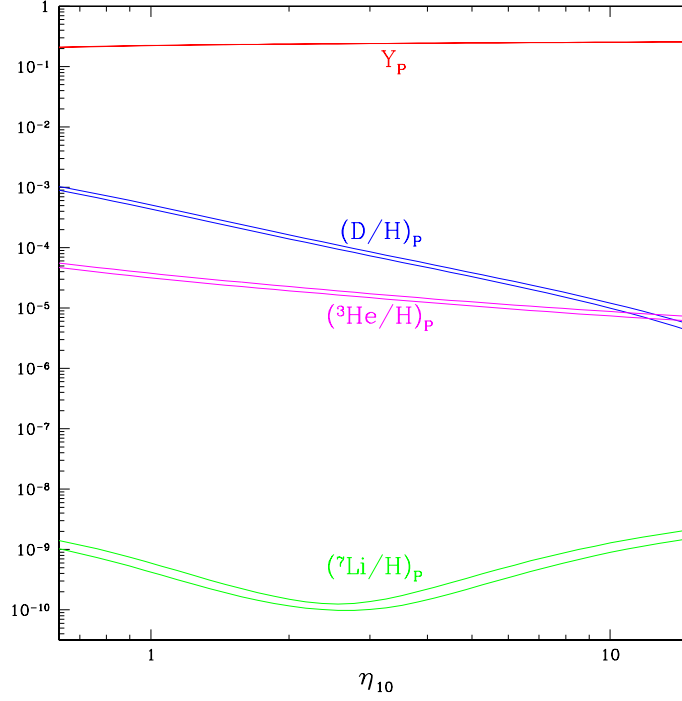
$$n \leftrightarrow p + e^- + \nu_e$$

are rapid enough to establish equilibrium between these various species. But when  $T$  falls below 1 MeV, the reaction rates become slower than the expansion rate of the Universe, and neutrons “freeze out”— they fall out of thermal equilibrium, as do the neutrinos. This means the equilibrium ratio of neutrons to protons at  $T \approx 1$  MeV is frozen into the expanding soup:  $n/p \approx 0.20 - 0.25$ . You all know that neutrons outside of an atomic nucleus are unstable particles and decay with a half-life of about 15 minutes. But before that happens there is a possible escape route:

$$n + p \leftrightarrow D + \gamma;$$

that is to say, a neutron can combine with a proton to make a deuterium nucleus and a photon. However, so long as the mean energy of particles and

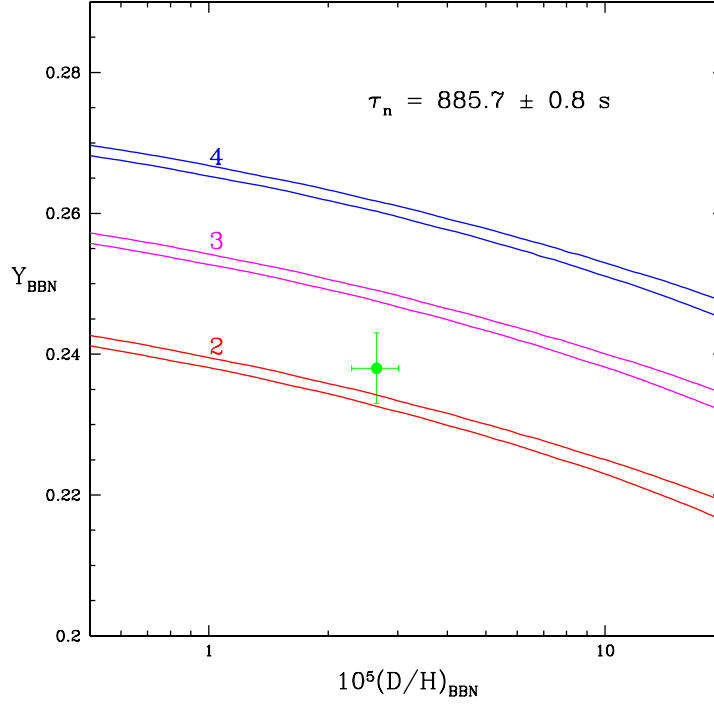
photons is greater than the binding energy of deuterium, about 86 Kev, the inverse reaction happens as well; as soon as a deuterium nucleus is formed it is photo-dissociated. This means that it is impossible to build up a significant abundance of deuterium until the temperature of the Universe has fallen below 86 KeV or, looking back at eq. 4.4, until the Universe has become older than about 2.5 minutes. Then all of the remaining neutrons are rapidly processed into deuterium. But the deuterium doesn't stay around for long either.



**Fig. 1.** The predicted abundances of the light isotopes as a function of  $\eta$  [18]. Here  $Y_p$  is the predicted mass fraction of helium and is based upon the assumption of three neutrino types. The widths of the bands show the theoretical uncertainty.

Given the temperature and particle densities prevailing at this epoch, there are a series of two-body reactions by which two deuterons combine to make  $\text{He}^4$  and trace amounts of lithium and  $\text{He}^3$ . These reactions occur at a rate which depends upon the overall abundance of baryons, the ratio of baryons to photons:

$$\eta = n_b/n_\gamma = 274 \Omega_b h^2 \times 10^{-10} \quad (4.5)$$



**Fig. 2.** The predicted abundance,  $Y_p$ , of helium (the mass fraction) as a function of the predicted deuterium abundance for two, three, and four neutrino types [18]. The point with error bars is the observed abundances of helium and deuterium.

So essentially all neutrons which survive until  $T = 86 \text{ KeV}$  become locked up in  $\text{He}^4$ . Therefore, the primordial abundance of helium depends primarily upon the expansion rate of the Universe: the faster the expansion (due, say, to more neutrino types or to a larger constant of gravity) the more helium. The abundance of remaining deuterium, however, depends upon the abundance of baryons,  $\eta$ : the higher  $\eta$  the less deuterium. This is why it is sometimes said [18] that the abundance of primordial helium is a good chronometer (it measures the expansion rate), while the abundance of deuterium is a good baryometer (it measures  $\Omega_b$ ). This is evident in Figs. 1 and 2 where we see first the predicted abundances of various light isotopes as a function of  $\eta$ , and secondly, the predicted abundance of He vs. that of deuterium for two, three and four neutrino types.

The determination of primordial abundances is not a straightforward matter because the abundance of these elements evolves due to processes occurring within stars (“astration”). In general, the abundance of helium increases

(hydrogen is processed to helium providing the primary energy source for stars), while deuterium is destroyed by the same process. This means that astronomers, when trying to estimate primordial abundances of deuterium or helium, must try to find pristine, unprocessed material, in so far as possible. One way to find unprocessed material is to look back at early times, or large redshift, before the baryonic material has been recycled through generations of stars. This can be done with quasar absorption line systems, where several groups of observers have been attempting to identify very shallow absorption lines of deuterium at the same redshift as the much stronger hydrogen Lyman alpha absorption line systems [19, 20, 21, 22]. It is a difficult observation requiring the largest telescopes; the lines identified with deuterium might be misidentified weak hydrogen or metal lines (incidentally, for an astronomer, any element heavier than helium is a metal). Taking the results of various groups at face value, the weighted mean value [18] is  $D/H \approx 2.6 \pm 0.3 \times 10^{-5}$ . Looking back at Fig. 1, we see that this would correspond to  $\eta = 6.1 \pm 0.6 \times 10^{-10}$  or  $\Omega_b h^2 = 0.022 \pm 0.003$ .

A word of caution is necessary here: the values for the deuterium abundance determined by the different groups scatter by more than a factor of two, which is considerably larger than the quoted statistical errors ( $\approx 25\%$ ). This indicates that significant systematic effects are present. But it is noteworthy that the angular power spectrum of the CMB anisotropies also yields an estimate of the baryon abundance; this is encoded in the ratio of the amplitudes of the second to first peak. The value is  $\Omega_b h^2 = 0.024 \pm 0.001$ . In other words, the two determinations agree to within their errors. This is quite remarkable considering that the first determination involves nuclear processes occurring within the first three minutes of the Big Bang, and the second involves oscillations of a photon-baryon plasma on an enormous scale when the Universe is about 500,000 years old. If this is a coincidence, it is truly an astounding one.

So much for the baryometer, but what about the chronometer—helium? Again astronomers are obliged to look for unprocessed material in order to estimate the primordial abundance. The technique of looking at quasar absorption line systems doesn't work for helium because the absorption lines from the ground state are far in the ultraviolet—about 600 Å for neutral helium and, more likely, 300 Å from singly ionized helium. This is well beyond the Lyman limit of hydrogen, where the radiation from the background quasar is effectively absorbed [23]. Here the technique is to look for He emission lines from HII regions (ionized gas around hot stars) in nearby galaxies and compare to the hydrogen emission lines. But how does one know that the gas is unprocessed? The clue is in the fact that stars not only process hydrogen into helium, but they also, in the late stages of their evolution, synthesize heavier elements (metals) in their interiors. Therefore the abundance of heavier elements, like silicon, is an indicator of how much nuclear processing the ionized gas has undergone. It is observed that the He abundance is correlated with the metal abundance; so the goal is to find HII regions with as low a metal

abundance as possible, and then extrapolate this empirical correlation to zero metal abundance [24, 25]. The answer turns out to be  $\text{He}/\text{H} \approx 0.24$ , which is shown by the point with error bars in Fig. 2.

This value is embarrassingly low, given the observed deuterium abundance. It is obviously more consistent with an expansion rate provided by only two neutrino types rather than three, but we know that there are certainly three types. Possible reasons for this apparent anomaly are:

1) Bad astronomy: There are unresolved systematic errors in determination of the relative He abundance in HII regions indicated by the fact that the results of different groups differ by more than the quoted statistical errors [18]. The derivation of the helium to hydrogen ratio from the observed  $\text{He}^+/\text{H}^+$  ratio requires some understanding of the structure of the HII regions. If there are relatively cool ionizing stars ( $T < 35000$  K) spatially separated from the hotter stars, there may be relatively less  $\text{He}^+$  associated with a given abundance of  $\text{H}^+$ . Lines of other elements need to be observed to estimate the excitation temperature; it is a complex problem.

2) New neutrino physics: There may be an asymmetry between neutrinos and anti-neutrinos (something like the baryon- antibaryon asymmetry which provides us with the observed Universe). This would manifest itself as a chemical potential in the Boltzmann equation giving different equilibrium ratios of the various neutrino species [26].

3) New gravitational physics: any change in the gravitational interaction which is effective at early epochs (braneworld effects?) could have a pronounced effect on nucleosynthesis. For example, a lower effective constant of gravity would yield a lower expansion rate and a lower He abundance. The standard minimal braneworld correction term, proportional to the square of the density [27], goes in the wrong direction.

It is unclear if the low helium abundance is a serious problem for the standard Big Bang. But it is clear that the agreement of the implied baryon abundance with the CMB determination is an impressive success, and strongly supports the assertion that the Hot Big Bang is the correct model for the pre-recombination Universe.

## 5 The post-recombination Universe: determination of $H_o$ and $t_o$

Certainly the most basic of the cosmological parameters is the present expansion rate,  $H_o$ , because this sets the scale of the Universe. Until a few years ago, there was a factor of two uncertainty in  $H_o$ ; with two separate groups claiming two distinct values, one near  $50 \text{ km s}^{-1}\text{Mpc}^{-1}$  and the other nearer  $100 \text{ km s}^{-1}\text{Mpc}^{-1}$ , and the errors quoted by both groups were much smaller than this factor of two difference. This points out a problem which is common in observational cosmology (or indeed, astronomy in general). Often the indicated statistical errors give the impression of great precision, whereas the

true uncertainty is dominated by poorly understood or unknown systematic effects. That was true in the Hubble constant controversy, and there is no less reason to think that this problem is absent in modern results. I will return to this point several times below.

The great leap forward in determination of  $H_o$  came with the Hubble Space Telescope (HST) program on the distance scale. Here a particular kind of variable stars—Cepheid variables—were observed in twenty nearby spiral galaxies. Cepheids exhibit periodic variations in luminosity by a factor of two on timescales of 2-40 days. There is a well-determined empirical correlation between the period of Cepheids and their mean luminosity—the longer the period the higher the luminosity. Of course, this period-luminosity relation must be calibrated by observing Cepheids in some object with a distance known by other techniques and this remains a source of systematic uncertainty. But putting this problem aside, the Hubble Space telescope measured the periods and the apparent magnitudes, without confusion from adjacent bright stars, of a number of Cepheids in each of these relatively nearby galaxies, which yielded a distance determination (eq. 3.9). These galaxies are generally too close (less than 15 Mpc) to sample the pure Hubble flow—the Hubble flow on these scales is contaminated by random motion of the galaxies and systematic cosmic flows—but these determinations do permit a calibration of other secondary distance indicators which reach further out, such as supernovae type Ia (SNIa) and the Tully-Fisher relation (the observed tight correlation between the rotation velocities of a spiral galaxies and their luminosities). After an enormous amount of work by a number of very competent astronomers [28], the answer turned out to be  $h = 0.72 \pm .10$

As I mentioned there is the known systematic uncertainty of calibrating the period-luminosity relation, but there are other possible systematic effects that are less well-understood: How can we be certain that the period-luminosity relation for Cepheids is the same in all galaxies? For example, is this relation affected by the concentration of elements heavier than helium (the metallicity)? In view of such potential problems, other more direct physical methods, which by-pass the traditional “distance ladder” are of interest. Chief among these is the Sunyaev-Zeldovich (S-Z) effect which is relevant to clusters of galaxies [29]. The baryonic mass of clusters of galaxies is primarily in the form of hot gas, which typically exceeds the mass in the visible galaxies by more than a factor of two. This gas has a temperature between  $10^7$  and  $10^8$  K (i.e., the sound speed is comparable to the one-dimensional velocity dispersion of the galaxies) and is detected by satellite X-ray telescopes with detectors in the range of several KeV. The S-Z effect is a small change in the intensity of the CMB in the direction of such clusters due to Compton scattering of CMB photons by thermal electrons (classical electron scattering would, of course, produce no intensity change). Basically, CMB photons are moved from the Rayleigh-Jeans part of the black body spectrum to the Wien part, so the effect is observable as a spectral distortion of the black body spectrum in the

range of 100 to 300 GHz. It is a small effect (on the order of 0.4 milli Kelvin) but still 5 to 10 times larger than the intrinsic anisotropies in the CMB.

By measuring the amplitude of the S-Z effect one determines an optical depth

$$\tau = \sigma n_e l \quad (5.1)$$

where  $\sigma$  is the frequency dependent cross section,  $l$  is the path length, and  $n_e$  is the electron density. Because these same clusters emit X-rays via thermal bremsstrahlung, we may also determine, from the observed X-ray intensity, an emission measure:

$$E = n_e^2 l \quad (5.2)$$

Here we have two equations for two unknowns,  $n_e$  and  $l$ . (This is simplifying the actual calculation because  $n_e$  is a function of radius in the cluster.) Knowing  $l$  and the angular diameter of the cluster  $\theta$  we can then calculate the angular size distance to the cluster via eq. 3.8. Hence, the Hubble parameter is given by  $H_o = v/D_A$  where  $v$  is the observed recession velocity of the cluster. All of this assumes that the clusters have a spherical shape on average, so the method needs to be applied to a number of clusters. Even so biases are possible if clusters have more typically a prolate shape or an oblate shape, or if the X-ray emitting gas is clumpy. Overall, for a number of clusters [30] the answer turns out to be  $h = 0.6$ —somewhat smaller than the HST distance ladder method, but the systematic uncertainties remain large.

A second direct method relies on time delays in gravitational lenses [31]. Occasionally, a distant quasar (the source) is lensed by an intervening galaxy (the lens) into multiple images; that is to say, we observe two or more images of the same background object separated typically by one or two seconds of arc. This means that there are two or more distinct null geodesics connecting us to the quasar with two or more different light travel times. Now a number of these quasars are intrinsically variable over time scales of days or months (not periodic but irregular variables). Therefore, in two distinct images we should observe the flux variations track each other with a time delay. This measured delay is proportional to the ratio  $D_l D_s / D_{ls}$  where these are the angular size distances to the lens, the source, and the lens to the source. Since this ratio is proportional to  $H_o^{-1}$ , the measured time delay, when combined with a mass model for the lens (the main source of uncertainty in the method), provides a determination of the Hubble parameter. This method, applied to several lenses [32, 33], again tends to yield a value of  $h$  that is somewhat smaller than the HST value, i.e.,  $\approx 0.6$ . In a recent summary [34] it is claimed that, for four cases where the lens is an isolated galaxy, the result is  $h = 0.48 \pm .03$ , if the overall mass distribution in each case can be represented by a singular isothermal sphere. On the other hand, in a well-observed lens where the mass distribution is constrained by observations of stellar velocity dispersion [35], the implied value of  $h$  is  $0.75^{+.07}_{-.06}$ . Such supplementary observations are important because the essential uncertainty with this technique is in the adopted mass model of the lens.

It is probably safe to say that  $h \approx 0.7$ , with an uncertainty of 0.10 and perhaps a slight bias toward lower values, but the story is not over as S-Z and gravitational lens determinations continue to improve. This is of considerable interest because the best fit to the CMB anisotropies observed by WMAP implies that  $h = 0.72 \pm .05$  in perfect agreement with the HST result. With the S-Z effect and lenses, there remains the possibility of a contradiction.

With  $h = .70$ , we find a Hubble time of  $t_H = 14$  Gyr. Now in FRW cosmology, the age of the Universe is  $t_o = ft_H$  where  $f$  is a number depending upon the cosmological model. For an Einstein-de Sitter Universe (i.e.,  $\Omega_k = 0$ ,  $\Omega_Q = 0$ ,  $\Omega_m = 1$ )  $f = 2/3$  which means that  $t_o = 9.1$  Gyr. For an empty negatively curved Universe,  $f = 1$  which means that the age is the Hubble time. Generally, models with a dominant vacuum energy density ( $\Omega_Q \approx 1$ ,  $w \approx -1$ ) are older ( $f \geq 1$ ) and for the concordance model,  $f = 0.94$ . Therefore, independent determinations of the age of the Universe are an important consistency test of the cosmology.

It is reasonable to expect that the Universe should be older than the oldest stars it contains, so if we can measure the ages of the oldest stars, we have, at least, a lower limit on the age of the Universe. Globular star clusters are old stellar systems in the halo of our own galaxy; these systems are distributed in a roughly spherical region around the galactic disk and have low abundances of heavy elements suggesting they were formed before most of the stars in the disk. If one can measure the luminosity,  $L_u$ , of the most luminous un-evolved stars in a globular cluster (that is, stars still burning hydrogen in their cores), then one may estimate the age. That is because this luminosity is correlated with age: a higher  $L_u$  means a younger cluster. Up to five years ago, this method yielded globular cluster ages of  $t_{gc} \approx 14 \pm 2$  Gyr, which, combined with the Hubble parameter discussed above, would be in direct contradiction with the Einstein-de Sitter  $\Omega_m = 1$  Universe. But about ten years ago the Hipparcus satellite began to return accurate parallaxes for thousands of relatively nearby stars which led to a recalibration of the entire distance scale. Distances outside the solar system increased by about 10% (in fact, the entire Universe suddenly grew by this same factor leading to a decrease in the HST value for the Hubble parameter). This meant that the globular clusters were further away, that  $L_u$  was 20% larger, and the clusters were correspondingly younger:  $t_{gc} \approx 11.5 \pm 1.3$  Gyr. If we assume that the Universe is about 1 Gyr older than the globular clusters, then the age of the Universe becomes  $12.5 \pm 2$  Gyr [36] which is almost consistent with the Einstein-de Sitter Universe. At least there is no longer any compelling time scale argument for a non-zero vacuum energy density,  $\Omega_Q > 0$ . The value of accurate basic astronomical data (and what is more basic than stellar positions?) should never be underestimated.

A second method for determining the ages of stars is familiar to all physicists, and that is radioactive dating. This has been done recently by observations of a  $U^{238}$  line in a metal-poor galactic star (an old star). Although the iron abundance in this star is only 1/800 that of the sun, the abundances



of a group of rare earth metals known as r-process elements are enhanced. The r-process is rapid neutron absorption onto iron nuclei (rapid compared to the timescale for subsequent  $\beta$  decay) which contributes to certain abundance peaks in the periodic table and which occurs in explosive events like supernovae. This means that this old star was formed from gas contaminated by an even older supernova event; i.e. the uranium was deposited at a definite time in the past. Now  $U^{238}$  is unstable with a half life of 4.5 Gyr which makes it an ideal probe on cosmological times scales. All we have to do is compare the observed abundance of  $U^{238}$  to that of a stable r-process element (in this case osmium), with what is expected from the r-process. The answer for the age of this star (or more accurately, the SN which contaminated the gas out of which the star formed) is  $12.5 \pm 3$  Gyr, which is completely consistent with the globular cluster ages [37].

If we take  $0.6 < h < 0.7$ , and  $9.5 \text{ Gyr} < t_o < 15.5 \text{ Gyr}$  this implies that  $0.59 < H_o t_o < 1.1$ . This is consistent with a wide range of FRW cosmologies from Einstein-de Sitter to the concordance model. That is to say, independent measurements of  $H_o$  and  $t_o$  are not yet precise enough to stand as a confirmation or contradiction to the WMAP result.

## 6 Looking for discordance: the classical tests

### 6.1 The angular size test

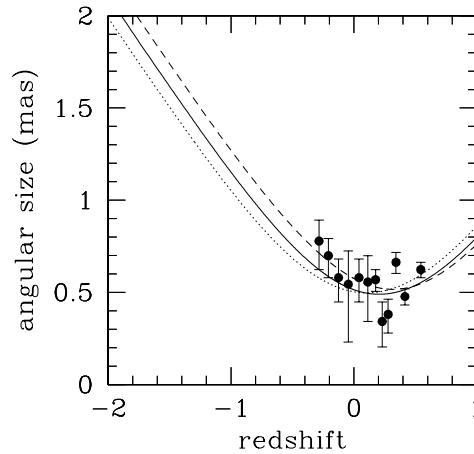
The first of the classical cosmological tests we will consider is the angular size test. Here one measures the angular size of a standard meter stick (hopefully) as a function of redshift; different FRW cosmologies make different predictions, but basically, for all FRW models  $\theta(z)$  first decreases as  $1/z$  (as would be expected in a Euclidean universe) and then increases with  $z$ . This is because the angular size distance is given by  $D_A = r/(1+z)$  but the radial comoving coordinate approaches a finite value as  $z \rightarrow \infty$ . The angular size distance reaches a maximum at a redshift between 1 and 2 and then decreases again.

When giant radio galaxies at large redshift were discovered in the 1960's there was considerable optimism that these could be used as an angular size cosmological probe. Radio galaxies typically have a double-lobe structure with the radio emitting lobes straddling the visible galaxy; these lobes can extend hundreds of kpc beyond the visible object. Such a linear structure may be oriented at any angle to the observer's line-of-sight, so one needs to measure the angular sizes of a number of radio galaxies in a given redshift bin and only consider the largest ones, i.e., those likely to be nearly perpendicular to the line-of-sight.

The result of all this work was disappointing. It appeared that the angular size of radio sources kept decreasing with redshift just as one would expect for a pure Euclidean universe [38]. The obvious problem, that plagues all classical tests, is that of evolution. Very likely, these radio galaxies are not standard

meter sticks at all, but that they were actually smaller at earlier epochs than now. This would be expected, because such objects are thought to result from jets of relativistic particles ejected from the nucleus of the parent galaxy in opposite directions. The jets progress through the surrounding intergalactic medium at a rate determined by the density of that medium, which, of course, was higher at larger redshift.

But there is another class of radio sources that would be less susceptible to such environmental effects: the compact radio sources. These are objects, on a scale of milli-arc-seconds, typically associated with distant quasars, that are observed with radio interferometers having global baselines. The morphology is that of a linear jet with lengths typically less than 30 or 40 pc, so these would presumably be emission from the jets of relativistic particles deep in the galactic nucleus near the central engine producing them. The intergalactic medium, and its cosmological evolution, would be expected to have no effect here [39].



**Fig. 3.** The median angular size vs. redshift (log-log plots) for 145 compact radio sources in 12 redshift bins. The curves are the three flat cosmological models: dashed,  $\Omega_A = 0.9$ ; solid,  $\Omega_A = 0.7$  (concordance), dotted,  $\Omega_A = 0.1$ . The physical size of the sources (20-40 pc) has been chosen for the best fit

The result of plotting the median angular size of about 150 of these sources as a function of redshift is shown on a log-log plot in Fig. 3 [40]. Also shown are the predicted relations for three flat cosmologies ( $\Omega_k = 0$ ) with  $\Omega_m = 0.9$ ,

0.3, 0.1, the remainder being in a cosmological constant (the middle curve is the concordance model). In each case the linear size of the compact radio sources was chosen to achieve the best fit to the data.

It is evident that the general property of FRW models (that the angular size of a standard meter stick should begin to increase again beyond a redshift of about 1.5) is present in this data. However, no statistical test or maximum likelihood analysis is necessary to see that all three models fit the data equally well. This is basically an imprecise cosmological test and cannot be improved, particularly considering that these objects may also evolve in some unknown way with cosmic time. Looking at the figure, one may notice that measurement of angular sizes for just a few objects at lower redshift might help distinguish between models. However, there are very few such objects at lower redshift, and these have a much lower intrinsic radio power than those near redshift one. It is dangerous to include these objects on such a plot because they are probably of a very different class.

## 6.2 The modern angular size test: CMB-ology

Although it is not my purpose here to discuss the CMB anisotropies, it is necessary to say a few words on the preferred angular scale of the longest wavelength acoustic oscillations, the “first peak”, because this is now the primary evidence for a flat Universe ( $\Omega_k = 0$ ). In Fig. 4 we see again the now very familiar plot of the angular power spectrum of anisotropies as observed by WMAP [42] (in my opinion, of all the WMAP papers, this reference provides the clearest discussion of the physics behind the peak amplitudes and positions). The solid line is the concordance model— not a fit, but just the predicted angular power spectrum (via CMBFAST [41]) from the  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$  model Universe with an optical depth of  $\tau \approx 0.17$  to the surface of last scattering. I must admit that the agreement is impressive.

I remind you that the harmonic index on the horizontal axis is related to angular scale as

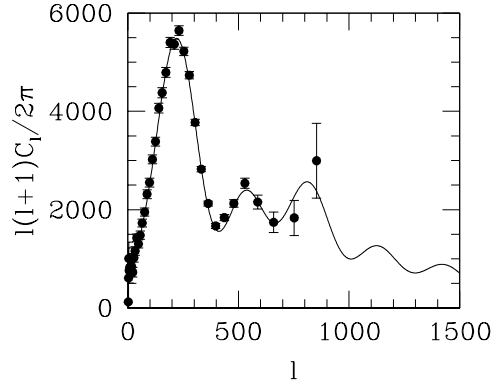
$$l \approx \pi/\theta \quad (6.1)$$

so the first peak, at  $l \approx 220$ , would correspond to an angular scale of about one degree. I also remind you that the first peak corresponds to those density inhomogeneities which entered the horizon sometime before decoupling (at  $z = 1000$ ); enough before so that they have had time to collapse to maximum compression (or expand to maximum rarefaction) just at the moment of hydrogen recombination. Therefore, the linear scale of these inhomogeneities is very nearly given by the sound horizon at decoupling, that is

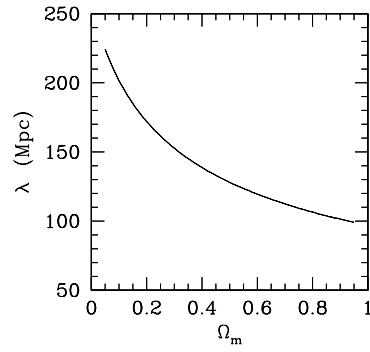
$$l_h \approx ct_{dec}/\sqrt{3} \quad (6.2)$$

where  $t_{dec}$  is the age of the Universe at decoupling.

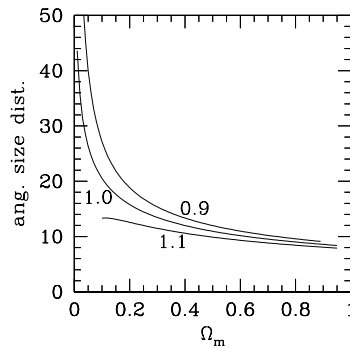
So one might say, the test is simple: we have a known linear scale  $l_h$  which corresponds to an observed angular scale ( $\theta \approx 0.014$  rad) so we can determine



**Fig. 4.** The angular power spectrum of CMB anisotropies observed by WMAP [42]. The solid line is not a fit but the is the concordance model proposed earlier [2]



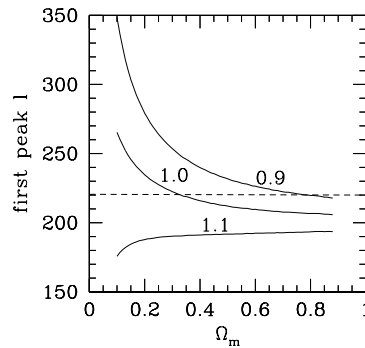
**Fig. 5.** The comoving linear scale of the perturbation corresponding to the first peak as a function of  $\Omega_m$



**Fig. 6.** The angular size distance (Gpc) to the last scattering surface ( $z = 1000$ ) as a function of  $\Omega_m$  for various values of  $\Omega_{tot}$

the geometry of the Universe. It is not quite so simple because the linear scale,  $l_h$  depends, via  $t_{dec}$  on the matter content of the Universe ( $\Omega_m$ ); basically, the larger  $\Omega_m$ , the sooner matter dominates the expansion, and the earlier decoupling with a correspondingly smaller  $l_h$ . This comoving linear scale is shown in Fig. 5 as a function of  $\Omega_m$  ( $\Omega_\Lambda$  hardly matters here, because the vacuum energy density which dominates today has no effect at the epoch of decoupling). Another complication is that the angular size distance to the surface of last scattering not only depends upon the geometry, but also upon the expansion history. This is evident in Fig. 6 which shows the comoving angular size distance (in Gpc) to the surface of last scattering as a function of  $\Omega_m$  for three values of  $\Omega_{tot} = \Omega_m + \Omega_\Lambda$  (i.e.,  $\Omega_k = 1 - \Omega_{tot}$ ). Note that the comoving angular size distance,  $D_A(1+z)$ , is the same as the radial comoving coordinate  $r$ .

We can combine Figs. 5 and 6 to plot the expected angular size (or harmonic index) of the first peak as a function of  $\Omega_m$  and  $\Omega_{tot}$ , and this is shown in Fig. 7 with the dashed line giving the observed  $l$  of the first peak. We see that a model with  $\Omega_{tot} \geq 1.1$  (a closed universe) is clearly ruled out, but it would be possible to have an open model with  $\Omega_{tot} = 0.9$  and  $\Omega_m = 0.8$  from the position of the first peak alone; the predicted peak amplitude, however, would be about 40% too low. The bottom line of all of this is that the *position* of the first peak does not uniquely define the geometry of the Universe because of a degeneracy with  $\Omega_m$  (I haven't mentioned the degeneracy with  $h$  taken here to be 0.72). To determine whether or not we live in a flat Universe we need an independent handle on  $\Omega_m$  and that is provided, in WMAP data, by the amplitudes of the first two peaks (the more non-baryonic matter, the deeper the forming potential wells, and the lower the amplitudes). From this



**Fig. 7.** The harmonic index expected for the first peak as a function of  $\Omega_m$  for various values of  $\Omega_{tot}$ .

it is found that  $\Omega_m \approx 0.3$ , and from Fig. 7 we see that the model Universe should be near flat ( $\Omega_{tot} \approx 1.0$ ). Of course if the Universe is near flat with  $\Omega_m = 0.3$  then the rest must be in dark energy; this is the indirect evidence from the CMB anisotropies for dark energy.

I just add here that the observed peak amplitudes (given the optical depth to  $z = 1000$  determined from WMAP polarization results [43]), is taken now as definitive evidence for CDM. However, alternative physics which affects the amplitude and positions of peaks (e.g. [3] could weaken this conclusion, as well as affect the derived cosmological parameters. Even taking the peak amplitudes as *prima facie* evidence for the existence of cold dark matter, it is only evidence for CDM at the epoch of recombination ( $z = 1000$ ) and not in the present Universe. To address the cosmic coincidence problem, models have been suggested in which dark matter transmutes into dark energy (e.g. [44]).

Now I turn to the direct evidence for dark energy.

### 6.3 The flux-redshift test: Supernovae Ia

Type I supernovae are thought to be nuclear explosions of carbon/oxygen white dwarfs in binary systems. The white dwarf (a stellar remnant supported by the degenerate pressure of electrons) accretes matter from an evolving companion and its mass increases toward the Chandrasekhar limit of about  $1.4 M_\odot$  (this is the mass above which the degenerate electrons become relativistic and the white dwarf unstable). Near this limit there is a nuclear detonation in the core in which carbon (or oxygen) is converted to iron. A nuclear flame

propagates to the exterior and blows the white dwarf apart (there are alternative models but this is the favored scenario [45]).

These events are seen in both young and old stellar populations; for example, they are observed in the spiral arms of spiral galaxies where there is active star formation at present, as well as in elliptical galaxies where vigorous star formation apparently ceased many Gyr ago. Locally, there appears to be no difference in the properties of SNIa arising in these two different populations, which is important because at large redshift the stellar population is certainly younger.

The peak luminosity of SNIa is about  $10^{10} L_{\odot}$  which is comparable to that of a galaxy. The characteristic decay time is about one month which, in the more distant objects, is seen to be stretched by  $1+z$  as expected. The light curve has a characteristic form and the spectra contain no hydrogen lines, so given reasonable photometric and spectroscopic observations, they are easy to identify as SNIa as opposed to type II supernovae; these are thought to be explosions of young massive stars and have a much larger dispersion in peak luminosity [46].

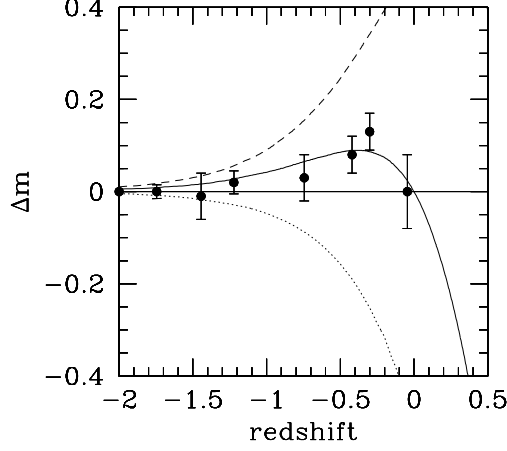
The value of SNIa as cosmological probes arises from the high peak luminosity as well as the observational evidence (locally) that this peak luminosity is the sought-after standard candle. In fact, the absolute magnitude, at peak, varies by about 0.5 magnitudes which corresponds to a 50%-60% variation in luminosity; this, on the face of it, would make them fairly useless as standard candles. However, the peak luminosity appears to be well-correlated with decay time: the larger  $L_{peak}$ , the slower the decay. There are various ways of quantifying this effect [46], such as

$$M_B \approx 0.8(\Delta m_{15} - 1.1) - 19.5 \quad (6.3)$$

where  $M_B$  is the peak absolute magnitude and  $\Delta m_{15}$  is the observed change in apparent magnitude 15 days after the peak [47]. This is an empirical relationship, and there is no consensus about the theoretical explanation, but, when this correction is applied it appears that  $\Delta L_{peak} < 20\%$ . If true, this means that SNIa are candles that are standard enough to distinguish between cosmological models at  $z \approx 0.5$ .

In a given galaxy, supernovae are rare events (on a human time scale, that is), with one or two such explosions per century. But if thousands of galaxies can be surveyed on a regular and frequent basis, then it is possible to observe several events per year over a range of redshift. About 10 years ago two groups began such ambitious programs [48, 49]; the results have been fantastically fruitful and have led to a major paradigm shift.

The most recent results are summarized in [50]: at present, about 230 SNIa have been observed out to  $z = 1.2$ . The bottom line is that SNIa are 10% to 20% fainter at  $z \approx 0.5$  than would be expected in an empty ( $\Omega_{tot} = 0$ ) non-accelerating Universe. But, significantly, at  $z \geq 1$  the supernovae appear to become brighter again relative to the non-accelerating case; this should happen in the concordance model at about this redshift because it is here



**Fig. 8.** The Hubble diagram for SNIa normalized to an empty non-accelerating Universe. The points are binned median values for 230 supernovae [50]. The curves show the predictions for three flat ( $\Omega_{tot} = 1$ ) cosmological models: The dashed line is the model dominated by a cosmological constant ( $\Omega_{\Lambda} = 0.9$ ), the solid curve is the concordance model ( $\Omega_{\Lambda} = 0.7$ ), and the dotted curve is the matter dominated model ( $\Omega_{\Lambda} = 0.1$ ).

that the cosmological constant term in the Friedmann equation (eq. 3.7) first begins to dominate over the matter term. This result is shown in Fig. 8 which is a plot of the median  $\Delta m$ , the observed deviation from the non-accelerating case, in various redshift bins as a function of redshift (i.e., the horizontal line at  $\Delta m = 0$  corresponds to the empty universe). The solid curves show the prediction for various flat ( $\Omega_{tot} = 1$ ) models with the value of the cosmological term indicated. It is evident that models dominated by a cosmological term or by matter are inconsistent with the observations at extremely high levels of significance, while the concordance model agrees quite well with the observations.

It is also evident from the figure that the significance of the effect is not large, perhaps 3 or 4  $\sigma$  (quite a low level of significance on which to base a paradigm shift). When all the observed supernovae are included on this plot, it is quite a messy looking scatter with a minimum  $\chi^2$  per degree of freedom (for flat models) which is greater than one. Moreover the positive result depends entirely upon the empirical peak luminosity-decay rate relationship and, of course, upon the assumption that this relation does not evolve. So, before we



become too enthusiastic we must think about possible systematic effects and how these might affect the conclusions. These effects include:

1) Dust: It might be that supernovae in distant galaxies are more (or less) dimmed by dust than local supernovae. But normal dust, with particle sizes comparable to the wavelength of light, not only dims but also reddens (for the same reason, Rayleigh scattering, that sunsets are red). This is quantified by the so-called color excess. Remember I said that astronomers measure the color of an object by its B-V color index (the logarithm of a flux ratio). The color excess is defined as

$$E(B - V) = (B - V)_{obs} - (B - V)_{int} \quad (6.4)$$

where *obs* means the observed color index and *int* means the intrinsic color index (the color the object would have with no reddening). In our own galaxy it is empirically the case that the magnitudes of absorption is proportional to this color excess, i.e.,

$$A_V = R_V E(B - V) \quad (6.5)$$

where  $R_V$  is roughly constant and depends upon average grain properties. So assuming that the dust in distant galaxies is similar to the dust in our own, it should be possible to estimate and correct for the dust obscuration. Significantly [48], it appears that there is no difference between  $E(B-V)$  for local and distant supernovae. This implies that the distant events are not more or less obscured than the local ones.

2) Grey dust: It is conceivable (but unlikely) that intergalactic space contains dust particles which are significantly larger than the wavelength of light. Such particles would dim but not redden the distant supernovae and so would be undetectable by the method described above [51]. It is here that the very high redshift supernovae ( $z > 1$ ) play an important role. If this is the cause of the apparent dimming we might expect that the supernovae would not become brighter again at higher redshift.

3) Evolution: It is possible that the properties of these events may have evolved with cosmic time. As I mentioned above, the SN exploding at high redshift come from a systematically younger stellar population than the objects observed locally. Moreover, the abundance of metals was smaller in the earlier Universe than now; this evolving composition, by changing the opacity in the outer layers or the composition of the fuel itself could lead to a systematic evolution in peak luminosity. Here it is important to look for observational differences between local and distant supernovae, and there seem to be no significant differences in most respects, the spectrum or the light curve. There is, however, a suggestion that distant supernovae are intrinsically bluer than nearby objects [46]. If this effect is verified, then it could not only point to a systematic difference in the objects themselves, but could also have lead to an underestimate of the degree of reddening in the distant SN. It is difficult, in general, to eliminate the possibility that the events themselves were different in the past and that this could mimic the effect of a cosmological constant

[52]; a deeper theoretical understanding of the SNIa process is required in order to realistically access this possibility.

4) Sample evolution: The sample of SN selected at large redshift may differ from the nearby sample that is used, for example, to calibrate the peak luminosity-decline rate correlation. There does appear to be an absence, at large redshift, of SN with very slowly declining light curves— which is to say, very luminous SN that are seen locally. Perhaps a class of more luminous objects is missing in the more distant Universe due to the fact that these SN emerge from a systematically younger stellar population. One would hope that the luminosity-decline rate correlation would correct for this effect, assuming, of course, that this relation itself does not evolve.

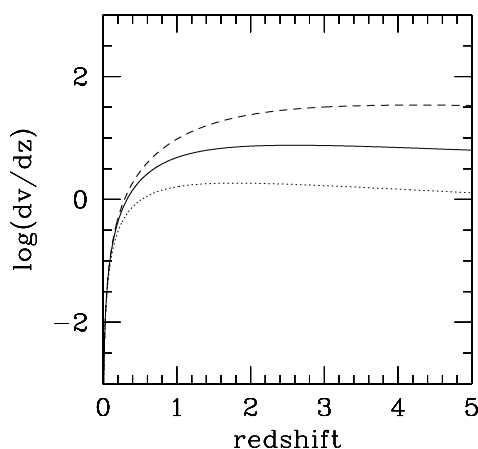
5) Selection biases: There is a dispersion in the luminosity-decline rate relationship, and in a flux-limited sample, one tends to select the higher luminosity objects. Astronomers call this sort of bias the “Malmquist effect” and it is always present in such observational data. Naively, one would expect such a bias to lead to an underestimate of the true luminosity, and, therefore an underestimate luminosity distance; the bias actually diminishes the apparent acceleration. But there is another effect which is more difficult to access: The most distant supernovae are being observed in the UV of their own rest frame. SNIa are highly non-uniform in the UV, and K-corrections are uncertain. This could introduce systematic errors at the level of a few hundredths of a magnitudes [50].

We see that there are a number of systematic effects that could bias these results. A maximum likelihood analysis over the entire sample [50], confirms earlier results that the confidence contours in  $\Omega_m$ - $\Omega_\Lambda$  space are stretched along a line  $\Omega_\Lambda = 1.4\Omega_m + 0.35$  and that the actual best fit is provided by a model with  $\Omega_m \approx 0.7$  and  $\Omega_\Lambda \approx 1.3$ — not the concordance model. Of course, if we add the condition that  $\Omega_{tot} = 1$  (a flat Universe) then the preferred model becomes the concordance model. In [50] it is suggested that this apparent deviation is due to the appearance of one or more of the systematic effects discussed above near  $z = 1$  at the level of 0.04 magnitudes.

The result that SNIa are systematically dimmer near  $z = 0.5$  than expected in a non-accelerating Universe is robust. At the very least it can be claimed with reasonable certainty that the Universe is not decelerating at present. However, given the probable presence of systematic uncertainties at the level of a few hundredths of a magnitude, it is difficult to constrain the equation of state ( $w$ ) of the dark energy or its evolution ( $dw/dt$ ) until these effects are better understood. I will just mention that lines of constant age,  $t_o H_o$ , are almost parallel to the best fit line in the  $\Omega_m$ - $\Omega_\Lambda$  plane mentioned above. This then gives a fairly tight constraint on the age in Hubble times [50]; i.e.  $t_o H_o = 0.96 \pm 0.4$ , which is consistent with the WMAP result. In a near flat Universe this rules out the dominance of matter and requires a dark energy term.

### 6.4 Number counts of faint galaxies

The final classical test I will discuss is that of number counts of distant objects— what radio astronomers call the  $\log(N)$ - $\log(S)$  test. Basically one counts the number of galaxies  $N$  brighter than a certain flux limit  $S$ . If we lived in a static Euclidean universe, then the number of galaxies out to distance  $R$  would be  $N \propto R^3$  but the flux is related to  $R$  as  $S \propto R^{-2}$ . This implies that  $N \propto S^{-3/2}$  or  $\log(N) = -3/2 \log(S) + \text{const} = 0.6m + \text{const}$ , where  $m$  is the magnitude corresponding to the flux  $S$ .



**Fig. 9.** The log of the incremental volume per incremental redshift (in units of the Hubble volume) as a function of redshift for the three flat cosmological models

But we do not live in a static Euclidean universe; we live in an evolving universe with a non-Euclidean geometry where the differential number counts probe  $dV(z)$ , the comoving volume as a function of redshift. In Fig. 9 we see  $\log(dV/dz)$  as a function of redshift for three different ( $\Omega_{tot} = 1$ ) cosmological models: the matter dominated Universe, the cosmological constant dominated Universe, and the concordance model. For small  $z$ ,  $dV/dz$  increases as  $z^2$  for all models as would be expected in a Euclidean Universe, but by redshift one, the models are obviously diverging, with the models dominated by a cosmological constant having a larger comoving incremental volume. Therefore if we can observe faint galaxies extending out to a redshift of one or two, we might expect number counts to provide a cosmological probe.

There is a long history of counting objects as a function of flux or redshift. Although cosmological conclusions have been drawn (see, e.g. [53]), the overall consensus is that this is not a very good test because the galaxy population evolves strongly with redshift. Galaxies evolve because stars evolve. In the past, the stellar populations were younger and contained relatively more massive, luminous stars. Therefore we expect galaxies to be more luminous at higher redshift. It is also possible that the density of galaxies evolves because of merging, as would be consistent with the preferred model of hierarchical structure formation in the Universe.

The distribution of galaxies by redshift can be used, to some extent, to break this degeneracy between evolution and cosmology. If we can measure the redshifts of galaxies with infrared magnitudes between 23 and 26, for example, that distribution will be skewed toward higher redshift if there is more luminosity evolution.

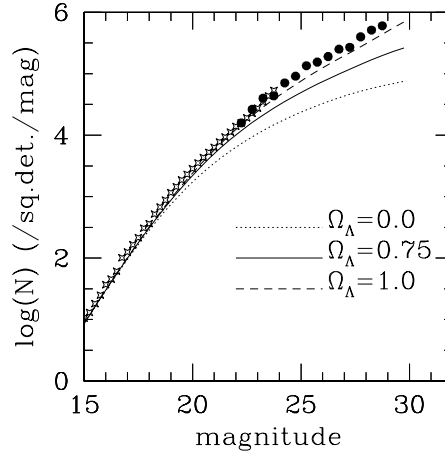
I have recently reconsidered the number counts of the faint galaxies in the Hubble Deep Fields, north and south [55, 56]. These are two separate small patches of empty sky observed with the Hubble Space Telescope down to a very low flux limit—about  $m_I = 30$  (the I band is a far red filter centered around 8000 angstroms). The differential number counts are shown by the solid round points in Fig. 10 where ground based number counts at fainter magnitudes are also shown by the starred points.

For this same sample of galaxies, there are also estimates of the redshifts based upon the galaxy colors—so called photometric redshifts [57]. In order to calculate the expected number counts and redshift distribution one must have some idea of the form of the luminosity function—the distribution of galaxies by redshift. Here, like everyone else, I have assumed that this form is given by the Schechter function [58]:

$$N(L)dL = N_o(L/L_*)^{-\alpha} \exp(-L/L_*)dL \quad (6.6)$$

which is characterized by three parameters:  $\alpha$ , a power law at low luminosities,  $L_*$  a break-point above which the number of galaxies rapidly decreases, and  $N_o$  a normalization. I take this form because the overall galaxy distribution by luminosity at low redshifts is well fit by such a law [59], so I am assuming that at least the form of the luminosity function does not evolve with redshift.

But when I consider faint galaxies at high redshift in a particular band I have to be careful to apply the K-correction mentioned above; that is, I must correct the observed flux in that band to the rest frame. Making this correction [60], but assuming no luminosity or density evolution, I find the differential number counts appropriate to our three flat cosmological models shown by the indicated curves in Fig. 10. We see that the predicted number counts all fall short of the observed counts, but that the cosmological constant dominated model comes closest to matching the observations. However, the distribution by redshift of HDF galaxies between I-band magnitudes of 22 and 26 is shown in Fig. 11 (this is obviously the cumulative distribution). Here we



**Fig. 10.** The solid points are the faint galaxy number counts from the Hubble Deep Fields (north and south [55, 56]) and the star shaped points are the number counts from ground based data. The curves are the no-evolution predictions from three flat cosmological models.

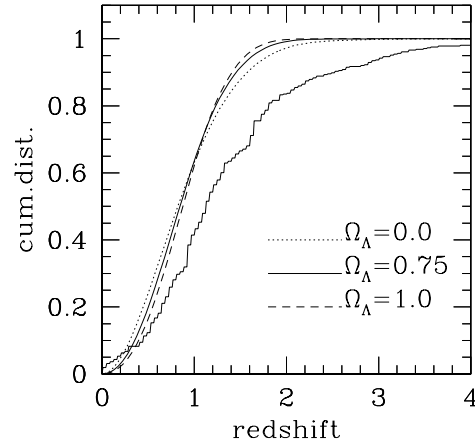
see that all three models seriously fail to match the observed distribution, in the sense that the predicted mean redshift is much too small.

This problem could obviously be solved by evolution. If galaxies are brighter in the past, as expected, then we would expect to shift this distribution toward higher redshifts. One can conceive of very complicated evolution schemes, involving initial bursts of star formation with or without continuing star formation, but it would seem desirable to keep the model as simple as possible; let's take a "minimalist" model for galaxy evolution. A simple one parameter scheme with the luminosity brightening proportional to the look-back time squared, i.e., every galaxy brightens as

$$\Delta M_I = q (H_o t_{lb})^2 \quad (6.7)$$

where  $q$  is the free parameter, can give a reasonable match to evolution models for galaxies [60]. (we also assume that all galaxies are the same— they are not divided into separate morphological classes). I choose the value of  $q$  such that the predicted redshift distribution most closely matches the observed distribution for all three models, and the results are shown in Fig. 12.

The required values of  $q$  (in magnitudes per  $t_H^2$ ) for the three cosmological models are:  $q = 2.0$  ( $\Omega_A = 1.0$ ),  $q = 3.0$  ( $\Omega_A = 0.7$ ), and  $q = 11.0$  ( $\Omega_A = 0.0$ ).

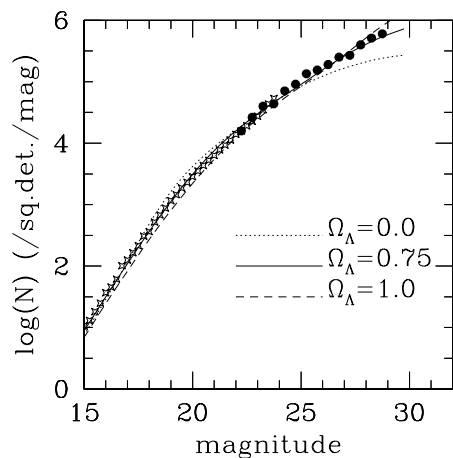


**Fig. 11.** The cumulative redshift distribution for galaxies between apparent I-band magnitudes of 23 and 26 (photometric redshifts from [57]). The curves are the predicted no-evolution distributions for the three cosmological models.

Obviously, the matter-dominated model requires the most evolution, and with this simple evolution scheme, cannot be made to perfectly match the observed distribution by redshift (this in itself is not definitive because one could always devise more complicated schemes which would work). For the concordance model, the required evolution would be about two magnitudes out to  $z = 3$ .

For these same evolutionary models, that is, with evolution sufficient to match the number counts, the predicted redshift distributions are shown in Fig. 13. Here we see that the model dominated by a cosmological constant predicts too many low redshift galaxies, the matter dominated model predicts too few, and the model that works perfectly is very close to the concordance model! Performing this operation for a number of flat models with variable  $\Omega_A$ , I find that  $0.59 < \Omega_A < 0.71$  to 90% confidence.

Now there are too many assumptions and simplifications to make this definitive. The only point I want to make is that faint galaxy number counts and redshift distributions are completely consistent with the concordance model when one considers the simplest minimalist model for pure luminosity evolution. One may certainly conclude that number counts provide no contradiction to the generally accepted cosmological model of the Universe (to my disappointment).

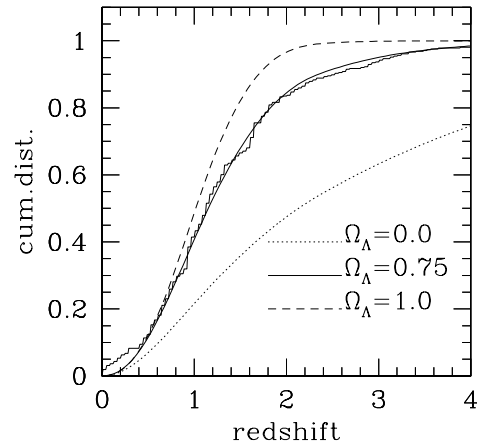


**Fig. 12.** As in Fig. 10 above the observed galaxy number counts and the predictions for the cosmological models with luminosity evolution sufficient to explain the number counts.

## 7 Conclusions

In these lectures I have been looking for discord, but have not found it. The classical tests return results for cosmological parameters that are consistent with but considerably less precise than those implied by the CMB anisotropies, given the usual assumptions. It is fair to say that the numbers characterizing the concordance model,  $\Omega_m \approx 0.3$ ,  $\Omega_\Lambda \approx 0.7$  are robust *in the context of the framework of FRW cosmology*. It is, in fact, the peculiar composition of the Universe embodied by these numbers which calls that framework into question.

Rather small changes in the assumptions underlying pure FRW cosmology (with only an evolving vacuum energy density in addition to more familiar fluids) can make a difference. For example, allowing  $w = -0.6$  brings the number counts and  $z$ -distribution of faint galaxies into agreement with a Universe strongly dominated by dark energy ( $\Omega_Q = 0.9$ ). The same also true of the high- $z$  supernovae observations [50]). Allowing a small component of correlated iso-curvature initial perturbations, as expected in braneworld cosmologies, can affect the amplitudes and positions of the peaks in the angular power spectrum of the CMB anisotropies [3], and therefore the derived cosmological parameters.



**Fig. 13.** The cumulative redshift distribution for galaxies between apparent i-band magnitudes of 22 and 26 (photometric redshifts from [57]). The curves are the predicted distributions for the three cosmological models with evolution sufficient to explain the number counts.

But even more drastic changes have been suggested. Certain braneworld scenarios, for example, in which 4-D gravity is induced on the brane [61] imply that gravity is modified at large scale where gravitons begin to leak into the bulk [62]. It is possible that the observed acceleration is due to such modifications and not to dark energy. More ad hoc modifications of General Relativity [5] have also been proposed because of a general unease with dark energy—proposals whereby gravity is modified in the limit of small curvature scalar. My own opinion is that we should also feel uneasy with the mysterious non-baryonic cold dark matter, because the only evidence for its existence, at present, is its gravitational influence; when the theory of gravity is modified to eliminate dark energy, it might also be found that the need for dark matter vanishes.

In general, more attention is being given to so-called infrared modifications of gravity (e.g. [63]), and this is a positive development. High energy modifications, that affect the evolution of the early Universe, are, as we have seen, strongly constrained by considerations of primordial nucleosynthesis (now, in combination with the CMB results). It is more likely that modifications play a role in the late, post-recombination evolution of the Universe, where the peculiarities of the concordance model suggest that they are needed. The fact



that the same rather un-natural values for the comparable densities of dark energy and matter keep emerging in different observational contexts may be calling attention to erroneous underlying assumptions rather than to the actual existence of these “ethers”.

Convergence toward a parameterized cosmology is not, without deeper understanding, sufficient reason for triumphalism. Rather, it should be a motivation to look more carefully at the possible systematic effects in the observations and to question more critically the underlying assumptions of the models.

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